MTH 201
Multivariable calculus and differential equations
Homework 7
Triple integrals and change of variables for double and triple integrals

Triple integrals

1. Evaluate each of the following triple integrals
   (a) $\iiint_R (x+y+z)\, dV$, where $R = [0,4] \times [0,3] \times [0,2]$.
   (b) $\iiint_R x\, dV$, where R is the region in the space bounded by $x = 0, y = 0, z = 2$, and the surface $z = x^2 + y^2$.
   (c) $\iiint_R 2x\, dV$, where R is the region under the plane $2x + 3y + z = 6$ that lies in the first octant (the first octant is the octant in which all three co-ordinates are positive).
   (d) $\iiint_R (\sqrt{x^2 + z^2})\, dV$, where R is the region bounded by $y = x^2 + z^2$ and the plane $y = 4$.
   (e) $\iiint_R 2x\, dV$, where R is the solid tetrahedron bounded by four planes $x = 0, y = 0, z = 0$, and $x + y + 2z = 4$.

2. Compute $\iiint_R xz\, dV$, where R is the solid tetrahedron with vertices $(0,0,0), (1,1,0), (0,1,0)$, and $(0,1,1)$.

3. Compute $\iiint_R x^2 e^y\, dV$, where R is the region below the parabolic cylinder $z = 1 - y^2$ and above the square $[-1,1] \times [-1,1]$ in the $xy-$plane.

Change of variables for double integrals

4. Determine the region that we get by applying the given transformation to the region D
   (a) D is the ellipse $x^2 + \frac{y^2}{36} = 1$ and the transformation is $x = u/2, y = 3v$.
   (b) D is the region bounded by $y = -x + 4, y = x + 1$, and $y = x/3 - 4/3$ and the transformation is $x = (u+v)/2, y = (u-v)/2$.

5. Evaluate the double integral $\iint_D e^{\frac{x+y}{x-y}}\, dA$, where D is the trapezoidal region with vertices $(1,0), (2,0), (0,-2)$, and $(0,-1)$.

6. Evaluate the double integral $\iint_D (x+y)\, dA$, where D is the trapezoidal region with vertices $(0,0), (5,0), (5/2, 5/2)$, and $(5/2, -5/2)$.

7. Evaluate $\iint_D (x^2 - xy + y^2)\, dA$, where D is the ellipse $x^2 - xy + y^2 = 2$ by changing variables $x = \sqrt{2}u - \sqrt{2/3}v$ and $y = \sqrt{2}u + \sqrt{2/3}v$.

8. Use change of variables $x = u^2 - v^2$ and $y = 2uv$ to evaluate the double integral $\iint_D y\, DA$, where D is the region bounded by the $X-$axis, the parabolas $y^2 = 4 - 4x, y^2 = 4 + 4x$, and $y \geq 0$.

Change of variables for triple integrals

9. (a) Convert the point $(-1, 1, \sqrt{2})$ from cartesian to cylindrical to spherical co-ordinates.

(b) Convert the point $(\sqrt{6}, \pi/4, \sqrt{2})$ from cylindrical to spherical co-ordinates.

10. Use change of variables to evaluate each of the following triple integrals

(a) $\iiint_R (x^2+y^2)\, dV$, where $R = \{(x, y, z) : -2 \le x \le 2, -\sqrt{4-x^2} \le y \le \sqrt{4-x^2}, \sqrt{x^2+y^2} \le z \le 2\}$.

(b) $\iiint_R e^{(x^2+y^2+z^2)^{3/2}}\, dV$, where R is the unit ball $R = \{(x, y, z) : x^2+y^2+z^2 \le 1\}$.

(c) $\iiint_R 16z\, dV$, where R is the upper half of the unit ball $\{(x, y, z) : x^2+y^2+z^2 \le 1\}$.

(d) $\int_{x=0}^{3} \int_{y=0}^{\sqrt{9-y^2}} \int_{z=\sqrt{x^2+y^2}}^{\sqrt{18-x^2+y^2}} (x^2+y^2+z^2)\, dzdydx$.

11. Evaluate $\iiint_R y\, dV$, where R is the region that lies below the plane $z = x+2$ above the $xy-$plane and between the cylinders $x^2+y^2 = 1$ and $x^2+y^2 = 4$.

12. Evaluate $\iint_R x^2\, dV$, where R is the solid region that lies within the cylinder $x^2+y^2 = 1$, above the plane $z = 0$, and below the cone $z^2 = 4(x^2+y^2)$.

13. Use spherical co-ordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2 = z$.